Fast algorithm for determining pupil and iris boundaries

Viktor Chigrinskiy

Moscow Institute of Physics and Technology

Advisor: Yuriy Efimov
Supervisor: Ivan Matveev

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Problem

Purpose

Building a fast algorithm to determine pupil and iris boundaries in eye image and approximate them by circles.

Proposal

It is proposed to perform three steps consequently: preprocess the eye image by morphological erosion and dilation, determine the pupil boundary via thresholding and determine the iris boundary using a density of the points distribution by their distances to the pupil center.

Problem solutions

Circular shortest path method

- I. A. Matveev. Circular shortest path as a method of detection and refinement of iris borders in eye image, 2011
- I. A. Matveev. Detecting precise iris boundaries by circular shortest path method, 2014

Hough transform and paired gradients

- I. A. Matveev, K. A. Gankin and A. N. Gneushev. Iris image segmentation based on approximate methods with subsequent refinements, 2014
- Y. S. Efimov and I. A. Matveev. Iris border detection using a method of paired gradients, 2015

Problem statement

Input

Monochromatic raster graphic image $\mathbf{I}_0$ size of $W \times H$, obtained by photographing wide-open eye in the near-infrared region by camera located approximately on the optical axis.

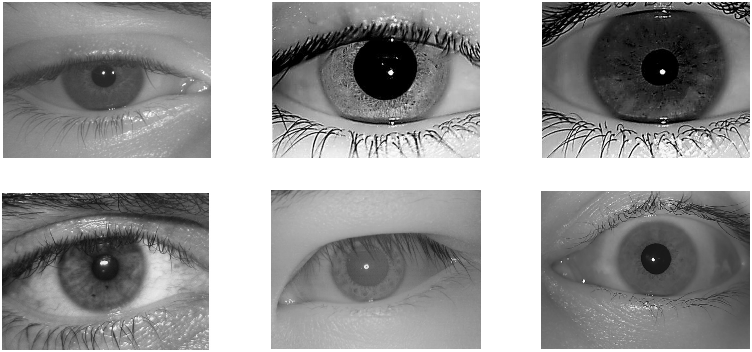
Output

Coordinates of centers and radiuses of two circles approximating pupil and iris boundaries: $\{\xi_{\text{pupil}}, \eta_{\text{pupil}}, \rho_{\text{pupil}}\}, \{\xi_{\text{iris}}, \eta_{\text{iris}}, \rho_{\text{iris}}\}$.

Expert data

For each image expert values of approximating circles is defined: $\tilde{\xi}_{\text{pupil}}, \tilde{\eta}_{\text{pupil}}, \tilde{\rho}_{\text{pupil}}, \tilde{\xi}_{\text{iris}}, \tilde{\eta}_{\text{iris}}, \tilde{\rho}_{\text{iris}}$

Input data examples



Quality criterion

Absolute error

Maximum among modules of parameters' deviations:

$$S = \max\left\{\left|\xi_{\text{pupil}} - \tilde{\xi}_{\text{pupil}}\right|, \left|\eta_{\text{pupil}} - \tilde{\eta}_{\text{pupil}}\right|, \left|\rho_{\text{pupil}} - \tilde{\rho}_{\text{pupil}}\right|, \left|\xi_{\text{iris}} - \tilde{\xi}_{\text{iris}}\right|, \left|\eta_{\text{iris}} - \tilde{\eta}_{\text{iris}}\right|, \left|\rho_{\text{iris}} - \tilde{\rho}_{\text{iris}}\right|\right\},$$

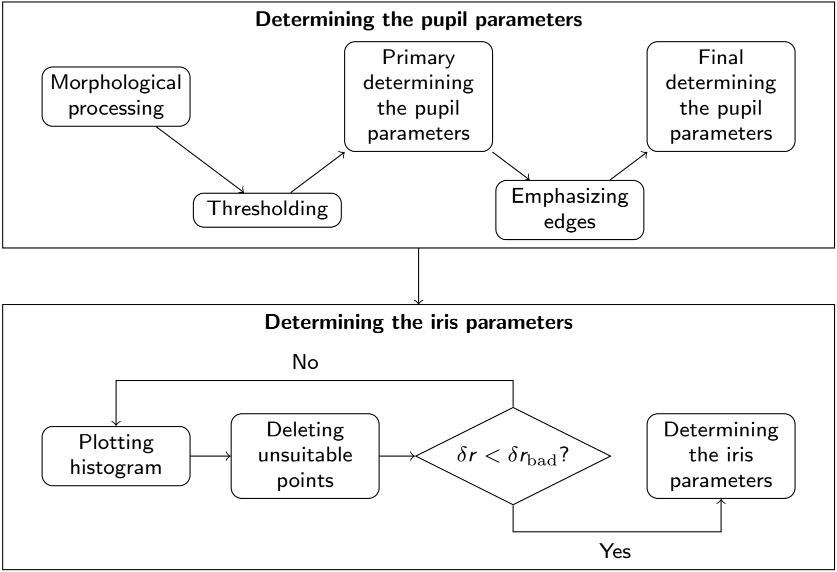
Relative error

The ratio of the absolute error to the iris radius: $e = \dfrac{S}{\tilde{\rho}_{\text{iris}}}$.

Quality criterion

The share of images on which the relative error does not exceed permissible value δ defined by expert.

Algorithm flowchart



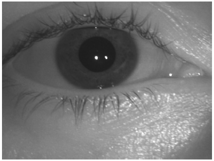
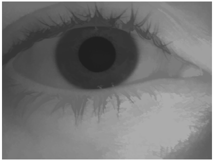
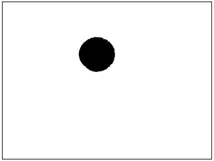
Mathematical morphology

- J. Serra. Image Analysis and Mathematical Morphology, 1983
- J. Serra. Image Analysis and Mathematical Morphology, vol. 2: Theoretical Advances, 1988

Canny edge detector

- J. F. Canny. A computational approach to edge detection, 1986

Primary determining pupil boundary

Morphological processing

Morphologically processed image $\mathbf{I}_{\mathrm{morph}}$ is obtained by consequent implementation of erosion and dilation of the initial image $\mathbf{I}_0$

Thresholding

Binary image is obtained according to the following rule:
$\mathbf{B}(\mathcal{T}; \xi, \eta) = [\mathbf{I}_{\mathrm{morph}}(\xi, \eta) \leqslant \mathcal{T}]$, where $\mathcal{T}$ is threshold value.

Choosing threshold value

The threshold value $\mathcal{T}$ is determined by the brute-force search which examines every pixel value in the image. Thresholded by the certain value τ image represented as a graph splits into N_{cc} connectivity components.

Effective radius of i-th component

$$r_{\mathrm{eff}}(\tau; i) = \max\{\xi_{\max}(\tau; i) - \xi_{\min}(\tau; i), \eta_{\max}(\tau; i) - \eta_{\min}(\tau; i)\}.$$

Quality of i-th component

$$q(\tau; i) = 1 - \left|1 - \frac{S(\tau; i)}{\pi r_{\mathrm{eff}}^2(\tau; i)}\right|.$$

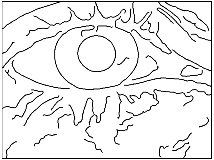
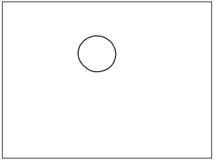
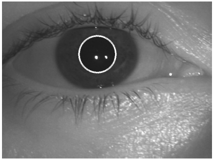
Quality of the threshold value τ

$$Q(\tau) = [N_{\mathrm{cc}}(\tau) > 0] \max_i q(\tau; i).$$

Threshold value $\mathcal{T}$

$$\mathcal{T} = \arg\max_\tau Q(\tau).$$

Final determining pupil boundary

Edge detection

The image of the edges $\mathbf{I}_{\mathrm{edge}}$ is obtained by applying the Canny edge detector to the morphologically processed image $\mathbf{I}_{\mathrm{morph}}$.

Pupil edge detection

The image of the pupil edges is obtained according to the following rule: $\mathbf{I}_{\mathrm{pupil}} = \left[\rho < \frac{5}{4}\rho_{\mathrm{pupil}}\right] \mathbf{I}_{\mathrm{edge}}$, where ρ is distance between the point and the pupil center.

Density of the edge points distribution

Real density

If assume that binary image is continuous set of points, it can be stated that the density of these points distribution by their distatnces to the pupil center exists. Let it be $f_{\mathrm{real}}(\rho)$

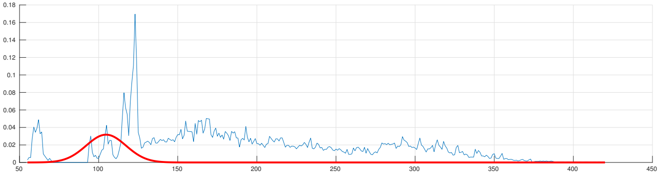
Effective density

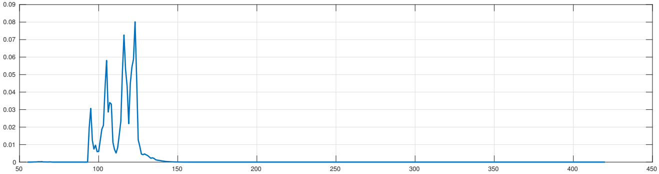
It is necessary to highlight the most probably iris radius because of the big amount of noise on a periphery of the image:

$$f(\rho) = \frac{f_{\mathrm{real}}(\rho)\nu(\rho_{\mathrm{pupil}};\rho)}{\int_{-\infty}^{+\infty} f_{\mathrm{real}}(\rho')\nu(\rho_{\mathrm{pupil}};\rho')d\rho'}$$

$$\nu(\rho_{\mathrm{pupil}};\rho) \sim \mathcal{N}\left(\mu,\sigma^2\right), \quad \mu = \frac{5}{2}\rho_{\mathrm{pupil}},\ \sigma = \frac{3}{10}\rho_{\mathrm{pupil}}$$

Highlighting most probable value





Iterative procedure for determining the iris boundary

Initialization

Those and only those edge points which can correspond to the iris boundary remain: $\mathbb{I}^{(0)}_{\text{iris}}(\xi,\eta) = \left[\frac{5}{4}\rho_{\text{pupil}} < \rho < 5\rho_{\text{pupil}}\right] \mathbb{I}_{\text{edge}}(\xi,\eta)$.

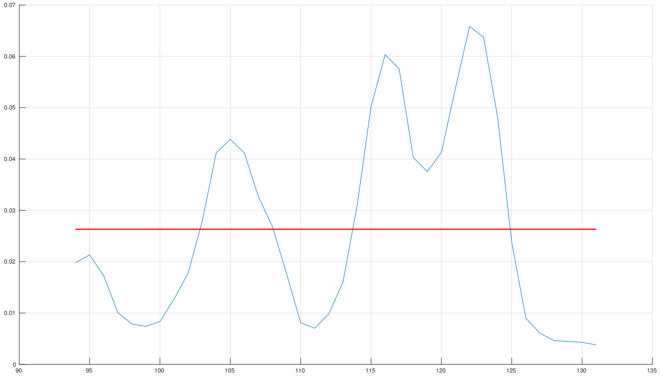
k-th step

$$\mathbb{I}^{(k)}_{\text{iris}} = \left[\forall\,\lambda \in [0,1] \quad f\left(\lambda\rho + (1-\lambda)\arg\max_{\rho} f(\rho)\right) > \frac{1}{\ell}\right] \mathbb{I}^{(k-1)}_{\text{iris}}.$$

Stopping criterion

At each step remaining points are approximated by circle and standard deviation $\delta r^{(k)}$ is calculated. The procedure repeats until this standard deviation exceeds the certain fixed value δr.

Iterative procedure for determining the iris boundary



Iterative procedure for determining the iris boundary

Computational approach for determining the iris boundary

In fact, images are not continuous, but discrete finite-size matrices. To build a numerical approximation of the real density of the points distribution it is proposed to round all ρ values to the nearest integer $\tilde{\rho}$.

Numerical approximation of the real density

$$\tilde{f}_{\mathrm{real}}(\tilde{\rho}) = \frac{n(\tilde{\rho})/2\pi\tilde{\rho}}{\sum n(\tilde{\rho}')/2\pi\tilde{\rho}'}.$$

Numerical approximation of the effective density

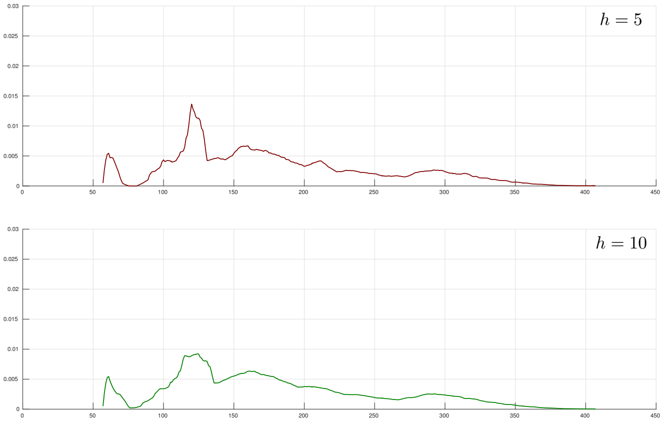
$$\tilde{f}(\tilde{\rho}) = \frac{\tilde{f}_{\mathrm{real}}(\tilde{\rho})\nu(\rho_{\mathrm{pupil}}; \tilde{\rho})}{\sum \tilde{f}_{\mathrm{real}}(\tilde{\rho}')\nu(\rho_{\mathrm{pupil}}; \tilde{\rho}')}.$$

Smoothing

Moving Average method

$$\tilde{f}_{\text{smooth}}(h;\tilde{\rho}) = \frac{1}{2h+1}\sum_{s=-h}^{h}\tilde{f}(\tilde{\rho}+s).$$

Smoothing



$h = 5$
$h = 10$
0.03
0.025
0.02
0.015
0.01
0.005
0
0 50 100 150 200 250 300 350 400 450

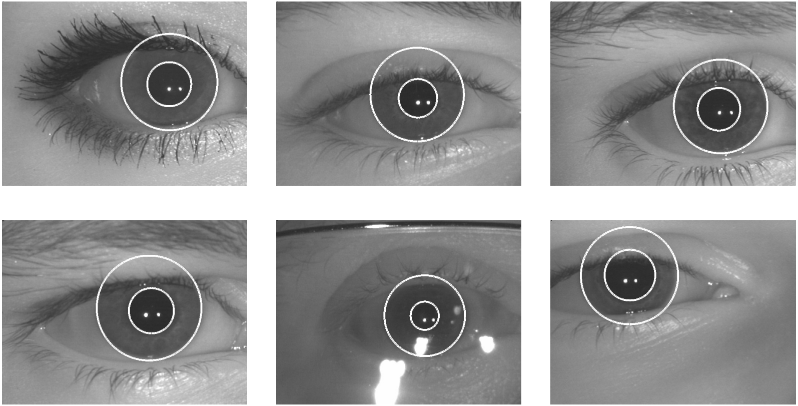
Require: the image $\mathbf{I}_0$.
Ensure: $\xi_{\text{pupil}}, \eta_{\text{pupil}}, \rho_{\text{pupil}}, \xi_{\text{iris}}, \eta_{\text{iris}}, \rho_{\text{iris}}$.

```
I_morph ← morphology(I_0)                          ▷ Morphological processing
for all values τ of pixel belonging I_morph do
    B(τ) ← binary(I_morph; τ)                      ▷ Thresholding by τ
    for all connectivity component i do
        obtain effective radius r_eff(τ; i)
        obtain quality of connectivity component q(τ; i)
    end for
    obtain quality of threshold Q(τ)
end for
choose threshold value 𝒯
obtain binary image B
choose connectivity component with the maximum q in the B
emphasize edges I_pupil of this component
ξ_pupil, η_pupil, ρ_pupil ← OLS(I_pupil)          ▷ Ordinary least squares
```

Pseudocode

$\mathbf{I}_{\text{edge}} \leftarrow \text{Canny}(\mathbf{I}_{\text{morph}})$ ▷ Canny edge detector
emphasize pupil edges $\mathbf{I}_{\text{pupil}}$
$\xi_{\text{pupil}}, \eta_{\text{pupil}}, \rho_{\text{pupil}} \leftarrow \text{OLS}(\mathbf{I}_{\text{pupil}})$
initialization of iris edges $\mathbf{I}^{(0)}_{\text{iris}}$
$k = 0$
$\delta r^{(0)} \leftarrow \text{OLS}(\mathrm{I}^{(0)}_{\text{iris}})$
while $\delta r^{(k)} > \delta r_{\text{bad}}$ **do**
    build approximation of the real density f_{real}
    build approximation of the effective density f
    smooth the approximated density f_{smooth}
    $k \leftarrow k + 1$
    do iteration step $\mathbf{I}^{(k)}_{\text{iris}}$
    $\delta r^{(k)} \leftarrow \text{OLS}(\mathbf{I}^{(k)}_{\text{iris}})$
end while
$\mathbf{I}_{\text{iris}} \leftarrow \mathbf{I}^{(k)}_{\text{iris}}$
$\xi_{\text{iris}}, \eta_{\text{iris}}, \rho_{\text{iris}} \leftarrow \text{OLS}(\mathbf{I}_{\text{iris}})$

Examples of the algorithm correct results

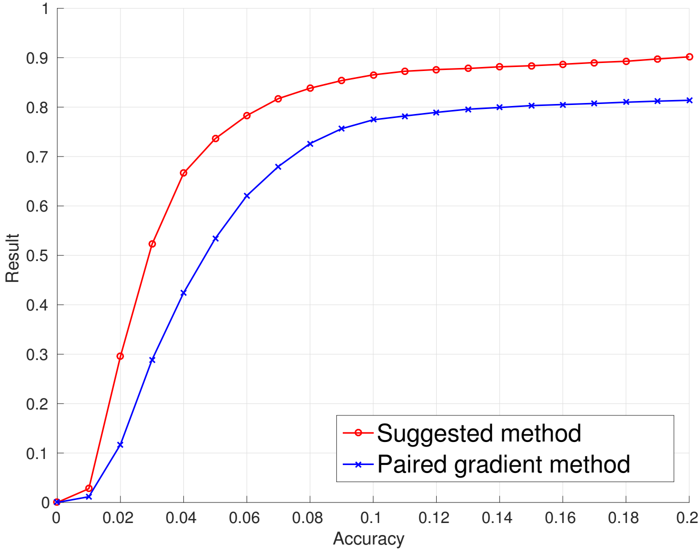


Accuracy and running time analysis

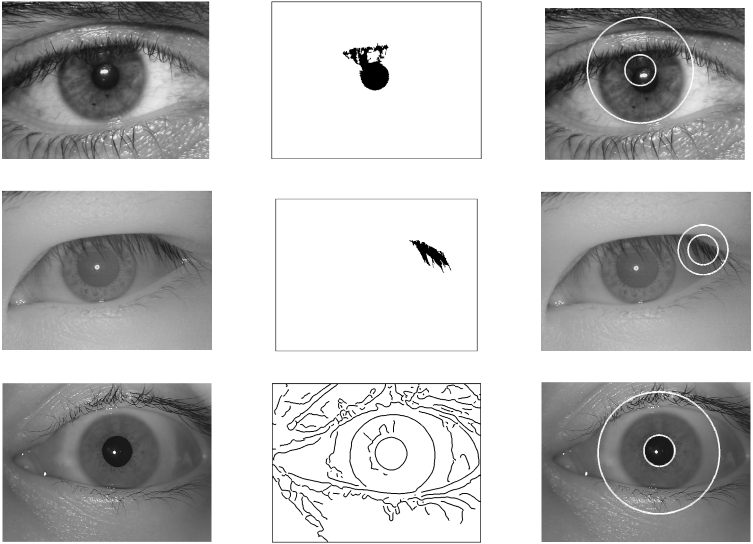
The accuracy results represent a percentage of images for those relative error doesn't exceed δ

δ	0.02	0.03	0.05	0.07	0.1	t, s
$h/\tilde{\ell}$	**Suggested method**					
0	25, 91	46, 68	71, 39	80, 82	87, 22	0, 246
0, 005	26, 21	46, 85	71, 47	81, 34	86, 96	0, 250
0, 01	28, 01	49, 72	73, 06	82, 11	87, 34	0, 253
0, 015	28, 61	50, 58	74, 05	82, 15	87, 26	0, 254
0, 02	**29, 56**	**52, 25**	**73, 62**	**81, 72**	**86, 53**	**0, 254**
0, 025	30, 12	52, 47	73, 14	81, 25	85, 89	0, 254
0, 03	29, 56	51, 91	72, 89	80, 48	84, 98	0, 254
—	**Paired gradient method**					
	11, 71	28, 87	53, 41	68, 00	77, 43	0, 432

Comparing with paired gradient method



Error analysis



Conclusion

- Fast algorithm for determining pupil and iris boundaries is built;
- operability of the algorithm is checked on the real data;
- suggested method compared with paired gradient method.