On the Dual and Inverse problems of scheduling problems with minimizing the maximum job penalty

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Problem

- 1 machine
- No preemtions
- Set of jobs $N = \{1, \dots, n\}$
- Release times $r_j, \ j \in N$
- Processing times $p_j, \ j \in N$
- Due dates $d_j, \ j \in N$

$$1 \mid r_j \mid \varphi_{\max}$$

$$1 \mid r_j \mid L_{max}$$

Problem

- $\pi = \{s_j | j \in N\}$ is a schedule
- $\Pi(N)$ is a set of all schedules from set N
- $S_j = S_j(\pi)$ is a start time of processing job $j \in N$
- $C_j = C_j(\pi)$ is a completion time of processing job $j \in N$

$$\mu^* = \min_{\pi \in \Pi(N)} \max_{k=1,\dots,n} \varphi_{j_k}(C_{j_k}(\pi)) \qquad (1)$$

Dual problem

$$\mu^* = \min_{\pi \in \Pi(N)} \max_{k=1,\dots,n} \varphi_{j_k}(C_{j_k}(\pi))$$

$$\nu^* = \max_{k=1,\dots,n} \min_{\pi \in \Pi(N)} \varphi_{j_k}(C_{j_k}(\pi)) \qquad (2)$$

Dual problem

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$$\nu_k = \min_{\pi \in \Pi(N)} \varphi_{j_k}(C_{j_k}(\pi)),\ k = 1, 2, \dots, n$$

Dual problem

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$$\nu_k = \min_{\pi \in \Pi(N)} \varphi_{j_k}(C_{j_k}(\pi)), \; k = 1, 2, \ldots, n$$

$$\nu^* = \max_{k=1,\ldots,n} \; \nu_k$$

Dual problem

$$\nu^* = \max_{k=1,\dots,n} \min_{\pi\in\Pi(N)} \varphi_{j_k}(C_{j_k}(\pi))$$

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$$\nu^* = \max_{k=1,\dots,n} \nu_k$$

Lemma

Let $\varphi_j(t), j = 1, 2, \dots, n$, be arbitrary non-decreasing penalty functions in the problem $1 \mid r_j \mid \varphi_{\max}$. Then, for all $k = 1, 2, \dots, n$, we have $\nu_n \geq \nu_k$, i.e., $\nu^* = \nu_n$.

Dual problem

$$\nu_k = \min_{\pi \in \Pi(N)} \varphi_{j_k}(C_{j_k}(\pi)),\ k = 1, 2, \dots, n$$

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Let $\varphi_j(t), j = 1, 2, \ldots, n$, be arbitrary non-decreasing penalty functions in the problem $1 \mid r_j \mid \varphi_{\max}$. Then, for all $k = 1, 2, \ldots, n$, we have $\nu_n \geq \nu_k$, i.e., $\nu^* = \nu_n$.

$$\nu_n = \min_{\pi \in \Pi(N)} \varphi_{j_n}(C_{j_n}(\pi))$$

Dual problem

Algorithm 1 for solving the dual problem to the problem $1 \mid r_j \mid \varphi_{\max}$

1. Construct the schedule $\pi^r = (i_1, i_2, \ldots, i_n)$, in which all jobs are sequenced according to non-decreasing release dates: $r_{i_1} \le r_{i_2} \le \cdots \le r_{i_n}$.

2. For $k = 1, 2, \ldots, n$, find the value $\varphi_{i_k}(C_{i_k}(\pi_k))$ for the schedule $\pi_k = (\pi^r \setminus i_k, i_k)$.

3. Find the value $\nu^* = \min\limits_{k=1,\ldots,n} \varphi_{i_k}(C_{i_k}(\pi_k))$ and the job i_k, which gives the value ν^*.

Estimating a lower bound

Theorem

Let $\varphi_j(t), j = 1, 2, \ldots, n$, be arbitrary non-decreasing penalty functions for the problem $1 \mid r_j \mid \varphi_{\max}$. Then the inequality $\mu^* \geq \nu^*$ holds.

Estimating a lower bound

Theorem

Let $\varphi_j(t), j = 1, 2, \ldots, n$, be arbitrary non-decreasing penalty functions for the problem $1 \mid r_j \mid \varphi_{\max}$. Then the inequality $\mu^* \geq \nu^*$ holds.

The obtained estimate can be efficiently used in constructing schemes of a branch and bound method for solving the problem $1 \mid r_j \mid \varphi_{\max}$, and for estimating the error of approximate solutions when the branch and bound algorithm stops without finding an optimal solution.

Algorithm for solving the problem $1 \mid r_j \mid L_{\max}$ with the branch and bound method based on the solution of the dual problem

1. Initial step

Let $\pi^* = \emptyset$. The list of instances contains the original instance $\{N, \tau, \nu, \emptyset\}$, where ν is the solution of the dual problem of this instance.

2. Main step

(a) From the list of instances, select an instance $\{N', \tau', \nu', \pi'\}$ with a minimal lower bound.

(b) Find the job $f = f(N', \tau')$ from the set N' with the smallest due date from the number of jobs ready for processing at time τ').

(c) Replace the instance $\{N', \tau', \nu', \pi'\}$ by the two instances $\{N_1, \tau_1, \nu_1, \pi_1\}$ and $\{N_2, \tau_2, \nu_2, \pi_2\}$ in the list of instances, where

– $N_1 = N' \setminus \{f\}, \tau_1 = \max\{r_f, \tau'\} + p_f, \nu_1$ is a solution of the dual problem for the instance $\{N_1, \tau_1\}, \pi_1 = (\pi', f)$; and

– $N_2 = N', \tau_2 = \tau'$, where for job f we use the updated value $r_f = \min_{j \in N' \setminus f} \{r_j(\tau_2) + p_j\}$ and ν_2 is a solution of the dual problem for the instance $\{N_2, \tau_2\}, \pi_2 = \pi'$.

(d) If, after completing this step of the algorithm, we obtain $\{\pi_1\} = N$, that is, all jobs are ordered, then $\pi^* = argmin\{L_{\max}(\pi_1, \tau), L_{\max}(\pi^*, \tau)\}$.

(e) Exclude all instances $\{N', \tau', \nu', \pi'\}$, for which $\nu' \geq L_{\max}(\pi^*, \tau)$

Algorithm for solving the problem $1 \mid r_j \mid L_{\max}$ with the branch and bound method based on the solution of the dual problem

3. Termination step

If the list of instances is empty, STOP, otherwise repeat the main step 2.

Computational results

Таблица: Results for Algorithm 2

$\|N\|$	Number of instance (set N)											
	1	2	3	4	5	6	7	8	9	10	11	12
10	73	59	17	9399	418	16	232	63	53	479	1062	160
11	98	179	4489	11965	1251	672	809	153	40	464	1060	125
12	673	69	828	5271	18866	2062	741	569	187	8235	1674	34
13	411	204	89	13850	6146	5331	684	521	112	18999	22748	42
14	23	101	*	95	4495	123	968	120	2137	51	23473	44
15	24	450	*	49188	1337	143	1170	619	513	52	23196	1166
16	25	370	*	252	373	630	572	1109	193	42	27634	66
17	641	2153	*	*	185	*	1520	1344	287	*	*	*
18	185	4521	*	*	186	*	450	1335	45	159	*	81
19	4612	25774	*	*	*	*	455	3681	618	*	*	132
20	286	*	*	*	*	*	596	6709	*	*	*	67

Thank you for your attention!

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